The Coefficient of Friction of the Pointe Shoe

and Implications for Current Manufacturing Processes

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Introduction

In 1832, the premiere of Filippo Taglioni's *La Sylphide*, starring his daughter, Marie, stunned Parisian audience members. Marie Taglioni's ability to balance momentarily on the tips of her toes intensified her portrayal of the ethereal sylph and represented the incorporation of pointework as a crucial choreographic element of ballet. From that moment on, pointework (the utilization of movements performed on the tips of one's toes) became inextricably associated with ballet. While pointework initially served as the ballerina's *tour de force*, it eventually became a fundamental technique for every female ballet dancer. Despite the widespread use of pointework in today's ballets, its allure and fascination remain.

The inception of and growing demand for pointework resulted in the development of pointe shoes. Originally slightly modified ballet slippers (tight-fitting shoes, made of fabric and leather, that allow movement in all directions but lack any supportive structures for the foot), pointe shoes evolved to provide support for the ballerina and to enable the execution of complicated steps on pointe. Advances in pointe shoe design were aided by the continuously reciprocal relationship between pointework and pointe shoe design. Advances in pointe shoe manufacturing allowed for advances in pointework technique, while increased demand for technical expertise required improvement in pointe shoe design. The interconnectedness between choreographic demands on the dancer and pointe shoe development provides an interesting perspective from which to examine the relationship between the physical design of the pointe shoe and the evolution of pointework technique.

Dance, like most art forms, has historically been slow to embrace and integrate scientific advances. Kenneth Laws, Professor Emeritus of Physics at Dickinson College and author of Physics and the Art of Dance, explains that dance, "as an aesthetic art form that depends largely on subjective judgments of quality rather than objectively quantifiable measures…may seem to be an activity not conducive to scientific scrutiny."[1] The clashing preferences of the artistic community and scientific world have translated into a strong reluctance against integration. In spite of this, recent contributions to understanding dance movement through scientific scrutiny have begun to shift this perspective.

Since the 1950s, interest in dance kinesiology has spurred an increased consideration for scientific applications to dance. Analysis of the musculoskeletal and biomechanical interactions involved in movement reveals the physical foundation of what a dancer's body is trained instinctively to sense. Much of the current scientific investigation involving dance focuses on injury prevention, incurred pressures on the body, biomechanics of the skeleton, and the magnitude of the forces exerted by muscles. The research specifically involving pointework focuses on the prevention of injury, the effect of using shoe orthoses with pointe shoes, and the determination of toe pressures while on pointe.[2] Some scientific articles address the relevant anatomy and injury-related kinematics of pointework.[3] One compares the strength and durability

[1] Kenneth Laws, "The Application of Physical Principles to Dance," in The Dancer as Athlete, edited by Caroline Shell (Champaign, Il: Human Kinetics Publishers, 1988), 123; Kenneth Laws, Physics and the Art of Dance, (New York: Oxford University Press, 2002).
[2] B. Cunningham et al., "A Comparative Mechanical Analysis of the Pointe Shoe Toe-Box: An In Vitro Study," American Journal of Sports Medicine 26(4) (1998): 556.
[3] Jarmo Ahonen, "Biomechanics of the Foot in Dance: A Literature Review," Journal of Dance Medicine and Science 12(3) (2008): 99-108; Jeffery A. Russell et al., "Clinical Anatomy and Biomechanics of the Ankle in Dance," Journal of Dance Medicine & Science 12(3) (July 2008): 75-82.

of various brands of pointe shoes.[4] These studies explain the physical repercussions of dancing on pointe. Very little research, however, has explored how the scientific consequences of dancing on pointe affect pointework technique.

The transition to dancing on pointe resulted in many physical implications for the ballerina. These include the greater lower body and foot strength required to support the dancer in an unstable position, an anatomical propensity to sickle (inward rotation of the foot so that the sole faces the central axis of the body), and a reduction in the contact area between the dancer's foot and the floor, which results in a decrease in friction and traction. Though each consequence is worthy of in-depth analysis, this study will focus on the latter consequence: the decreased contact area and traction between the foot of the dancer and the floor. My test of today's pointe shoes has determined the coefficients of friction for satin and canvas pointe shoe platforms. This value can be analyzed in relation to traction and turn deceleration and then related to specific developments in pointe technique.

Examining pointework evolution through the lens of a single physical attribute of pointe-shoe-use demonstrates that scientific research can help in understanding how pointe shoes function most effectively. In the sports world, athletic equipment manufacturers often make use of new research and advances in technology and physiology to improve the effectiveness of their products and safeguard the health of the athlete. One example of this is the development of the running shoe, which in only a few decades has changed from a simple sneaker to a sophisticated aerodynamic, foot-protecting performance necessity. Almost all pointe shoes, however, have maintained the use of traditional materials and manufacturing procedures. For footwear that radically affects the placement and alignment of the body, and allows for ever-increasing choreographic demands, most pointe shoes remain relics of past manufacturing traditions rather

[4] Cunningham et al., 551-561.

than state-of-the-art unions of technological innovation, safety, and aesthetic perfection. A growing interest in the application of science to dance sets the stage for a new outlook on pointe shoe design and manufacturing methods. The ability to apply the analysis of the frictional forces acting on contemporary pointe shoes to significant changes in pointe technique suggests that scientific research offers bright hopes for improving pointe shoe designs to benefit the dancer.

History and Anatomy of the Pointe Shoe

While the exact date is unknown, the first use of pointework occurred during the eighteenth century. In 1721-22, the Lincoln's Inn Fields in London billed a dance called the *Dutch Skipper* as a man dancing "on his toes."[5] Of the dance treatises from the eighteenth century, only one mentions toe dancing. In 1779, Gennaro Magri wrote about the dancer Antoine-Bomaventure Pitrot who lifted all of his weight onto his big toe.[6] Pitrot was praised for his supernatural appearance, a forewarning of the idolization of this quality during the Romantic era. Also at this time, male and female grotesque dancers (dancers who impersonated comic, popular, or grotesque characters) utilized pointework in the portrayal of some roles.[7] The French Revolution in 1789 caused a resurging interest in antiquity and an overnight change in ballet costuming from restrictive, corseted gowns to loose, semi-transparent tunics. The freedom of the now flowing, light costumes permitted women to perform more demanding movements. More importantly, the revolution produced a desire for theater to provide an escape from the hard realities of everyday life. This is evidenced by the enthusiastic reception of Charles Didelot's *Zéphire et Flore*; produced in London in 1796. His elaborate flying spectacles allowed the lead

[5] Sandra Noll Hammond, "Searching for the Sylph: Documentation of Early Developments in Pointe Techninque," Dance Research Journal, 19(2) (Winter 1987-1988): 29.
[6] Hammond, 29.
[7] Deborah Jowitt, Time and the Dancing Image (New York: William Morrow Company, 1988), 38.

ballerina to dance with only her toes grazing the stage.[8] The illusion of weightlessness continued to amaze audiences in St. Petersburg in 1804 and Paris in 1815.

The use of pointework gradually spread during the early nineteenth century. Geneviève Gosselin, a ballerina at the Paris Opera from 1806 to her death in 1818, was documented as dancing on her toes in 1815.[9] Marie Taglioni watched Amalia Brugnoli dance on her toes in Vienna in 1823, but disliked the effort and contorted motions required to rise onto pointe.[10] At this time, dancers still wore the "supple cothurns" or "glove-fitting slippers" that replaced heeled shoes in the 1790s.[11] While the footwear of the dancers provided no physical support for balancing on the toes, the documentation of isolated occurrences of pointe use sets the stage for its formal recognition in the 1830s.

Many critics acknowledge Marie Taglioni's 1832 performance in *La Sylphide* as the critical moment when pointework became a defining element of female ballet dancing. This moment also marked the beginning of the Romantic period in ballet. The Romantic movement spread across Europe and affected all art forms. In dance it triggered a rejection of the mythological themes of eighteenth-century ballets. Instead, female dancers impersonated ethereal creatures, who would draw the human hero from the physical world to their idyllic dwellings in nature. The typical Romantic ballet included a *ballet blanc* showcasing the light, airy quality of the female dancers' movements. Pointework greatly contributed to the ballerina's apparent exemption from the laws of gravity. The fascination with the light, supernatural quality

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[8] Jowitt, 38.
[9] Jowitt, 38.
[10] Jowitt, 38.
[11] Marian Hannah Winter, The Pre-Romantic Ballet (New York: Dance Horizons, 1975), 3.

of the dancers was a reaction to the public's dissatisfaction with the limitations of the human world.[12]

Reference to pointework in dance manuals of the 1830s further confirms its incorporation into contemporary dance style. Léon Michel, who had danced at the Paris Opera, wrote, in 1830 and 1831, about dancing *sur les orteils*, or on full pointe, and offers advice on how the dancer is meant to rise to full pointe.[13] A former ballet master at the *Académie Royale de Danse* in Paris, E.A. Théleur in his 1830 treatise, Letters on Dancing, referred to and illustrated pointework without addressing the subject directly. In his description of the "stations" (positions) of the body, he asserts that the positions are completed "on the balls of the feet or on the pointes of the toes."[14] Several stations and arm positions are illustrated showing female dancers standing on the tips of their toes.[15] He also describes exercises that incorporate rising to full pointe:

> To gain strength on the pointes of the toes, the strength of the joints of the great toes should be added to that of the ankles, keeping the joints of the toes perfectly straight from the commencement of the movement.[16]

The presence, in Theleur's treatise, of instructions for dancing on pointe demonstrates his interest in promoting the use of pointework. Dance manuals, however, inherently document existing dance methods. Therefore, Théleur's treatise suggests that pointework had become a significant consideration in ballet dancing by 1830. Strangely, no mention of pointework exists in Carlo Blasis' books on ballet technique written in the same period, Traité Élémentaire Théorique et Pratique de l'art de la Danse (1820) and The Code of Terpsichore (1828-1830).[17]

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[12] Jowitt, 40.
[13] Hammond, 28.
[14] E. A. Théleur, "Letters on Dancing, Reducing this Elegant and Healthful Exercise to Easy Scientific Principles," Studies in Dance History, 2(1) (Fall/Winter 1990): 15, 18, 20, 22, 24.
[15] Théleur, Plates 7, 10, 12, 14, 16, 18, 20.
[16] Théleur, 55.
[17] Hammond, 27.

As dancing on pointe grew in popularity during the 1830s and 1840s, the pointework vocabulary gradually expanded. To perform more complicated steps on pointe, ballerinas required more supportive footwear. According to Ivor Guest, a dance historian who specializes in ballet of the eighteenth- and nineteenth-centuries, ballerinas during the emergence of pointework continued to wear "light satin slippers with flexible leather soles and ribbons."[18] Ballerinas resorted to strengthening their footwear by darning (a sewing method typically used for repairing holes in fabric) the "tip and then inserting wadding to protect the toes."[19] By the middle of the century, however, ballerinas had begun to work with shoemakers to design a stronger shoe that was more conducive to dancing on pointe. As Deborah Jowitt, dance critic and author of Time and the Dancing Image, explains:

> Years of interchange between ambitious dancers and ingenious shoemakers had gradually transformed the pointe shoe from a thin slipper reinforced by darning and transformed to something with a steel shank and a stiffened box of a toe.[20]

The majority of the changes in pointe shoe design occurred in Italy, the center of pointe shoe manufacturing and home to the era's leading ballet school affiliated with the La Scala Theater.

The revamping of the pointe shoe allowed the ballerina to progress from standing on pointe for a few moments to completing difficult choreography on pointe. In a review of Jules Perrot's 1858 production of *Eoline*, Théophile Gautier raved about Amalis Ferraris's prowess in dancing on pointe: "A triple pirouette on pointe drew gasps of amazement….She not only steps on her pointes, but she jumps on them."[21] The support of the new shoe design enabled the ballerina to complete a variety of virtuousic steps. After the 1860s, dancers increasingly wore

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[18] Ivor Guest, "Costume and the Nineteenth Century Dance," in Designing for the Dancer, edited by Roy Strong, (London: Eltron Press, 1981), 54.
[19] Guest, 54.
[20] Jowitt, 247.
[21] Marian Horosko, "If the Shoe Fits…Part II," Dance Magazine, May 1986, 99; Jowitt, 247.

blocked shoes, which had toes stiffened by glue and strong interlacing.[22] In 1866, Théophile Gautier noted that "[the] sole, which is very hollowed out in the centre, does not reach the tip of the foot but ends squarely, leaving about two finger-breadths of material projecting."[23] He went on to explain that the dancer must strengthen the joint of the slipper with darning. The public's fascination with and demand for pointework transformed pointe shoe manufacturing into a specialized industry. Stronger pointe shoes enabled ballerinas to accomplish more demanding movements on pointe.

The latter half of the nineteenth century witnessed further development of and demand for pointe technique. The dance style evolved to incorporate pointework whenever possible. As Deborah Jowitt explains, "[slow] adagios on the whole foot all but disappeared; the public wanted to see pointework."[24] With the dissemination of pointe, variations in its technique developed. The French school taught how to relevé, the motion of rising onto pointe, as a continuous roll-up. The Italian and Russian schools preferred a spring to pointe.[25] Despite the emergence of national preferences, ballerinas everywhere remained most admired for their expertise on pointe.

At the end of the nineteenth century the epicenter of the dance world shifted from Italy to Russia. Dancers trained at the Imperial Theater under chief dance master and choreographer Marius Petipa (at the Imperial Ballet until 1904). In the 1880s and 1890s, Petipa drew inspiration from the touring Italian ballerinas whose stronger pointe shoes allowed them to perform more challenging steps on pointe. Copying shoe manufacturing techniques and borrowing pointework movements from the visiting dancers, Petipa began choreographing works that demanded much

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[22] Robert Greskovic, "Pointe Work," International Encyclopedia of Dance .
[23] Guest, 55.
[24] Jowitt, 248.
[25] Greskovic.

more virtuosity in pointework. His choreography of *The Sleeping Beauty* in 1890 included a "Pas d'Action" in Act I, called "The Rose Adagio," that required the ballerina to balance on pointe for more than a minute at a time while four suitors took turns supporting her. In Petipa's Act III choreography for *Swan Lake* in 1895, Odile had to complete 32 fouettés on pointe, a feat performed by the La Scala-trained virtuosa Pierina Legnani. Petipa's choreography stretched the ballerina's technical repertoire to new heights and demands. Without the support of a blocked pointe shoe with a broader and firmer base, these movements would not have been feasible.

The incorporation of pointework into ballets almost disappeared at the beginning of the twentieth century. Choreographers strongly critiqued the traditional ballerina, and began to eliminate pointework from their productions. Michel Fokine, a teacher at the Imperial Ballet School in St. Petersburg and choreographer for Serge Diaghilev, used pointework sparingly. In *The Firebird,* choreographed in 1910, only the title character danced on pointe while *Shéhérazade* eliminated pointework altogether. Later works by Diaghilev's Ballets Russes company showed a resurgence of the technique and experimentation with the qualities expressed through pointework. In 1923 and 1924, Bronislava Nijinska choreographed several works for Diaghilev, including *Les Noces* and *Les Biches*. Both of these ballets used pointework, although not as an emphasis on the lightness of the dancers. In *Les Noces*, the shoes are used almost as percussive instruments, emphasizing the dancer's heavy contact with the floor.[26]

Around the turn of the century, several pointe shoe manufacturing companies were founded, including Capezio in 1889 and Freed of London in 1929, which initiated the practice of designing shoes to fit a specific foot.[27] Previously, dancers had to force their feet into a uniformly narrow pointe shoe, regardless of their foot shape. The establishment of new

[26] Bronislava Nijinska, "Creation of 'Les Noches': Bronislava Nijinksa." translated and introduced by Jean M. Serafetinides and Irina Nijinska, Dance Magazine, December 1974, 59.
[27] Michele Attfield, "Evolution of the Pointe," Dancing Times, July 2003, 25.

companies allowed a professional dancer to consult with a specific "maker" and design a shoe that fit her feet. Each maker specialized in a certain style of shoe, but was able to cater the construction to individual desires. Most manufacturers produced "turned" shoes: the shoe was constructed inside-out from the sole up and then turned right-side-out at the end.[28] Generally, each shoe was molded to a "last" that represented the shape of a foot. The basic design of the pointe shoe changed little throughout these years, but the ability to tailor this to the individual foot allowed for more comfortable and supportive shoes.

George Balanchine, after serving as ballet master for Diaghilev's Ballet Russes, founded the School of American Ballet in 1934. By the 1960s, his ideas had revolutionized modern pointework. He insisted that all female dancers perform in pointe shoes and wear them in class and rehearsal. Through his teaching, he emphasized the importance of full turnout and the roll-up relevé. He believed that rolling through the foot strengthened the lower body muscles and allowed for quicker articulation of steps. He also had clear ideas about the aesthetic look and sound of pointe shoes.[29] He desired dramatically tapered toe boxes that elongated the legs and believed that steps performed using pointe shoes should not be audible. Balanchine's choreography used an enormous amount of pointework and pushed the technique to new levels of difficulty and complexity. Despite these radical changes in pointework technique, pointe shoe construction remained relatively unchanged, except for a slight widening of the box.

Throughout the twentieth century, most companies continued to manufacture pointe shoes using the traditional "turned" method with two significant exceptions. In the 1970s, manufacturers began to offer a greater range of shoe styles and sizes allowing the dancer to fine-

[28] Dan Terlizzi, interview by Glenna Clifton, Fairlawn, NJ, 2 October 2009. Terlizzi is the Company Director of Capezio Ballet Makers.
[29] Suki Schorer, Suki Schorer on Balanchine Technique, (New York: Random House, 1999), 7, 9.

tune the look and feel of her shoes within the basic frame of the shoe style and materials.[30] Additionally, in the past two decades, a select few companies have begun to explore new materials that could prolong the life of pointe shoes and help prevent dancer injuries. A secondary industry has developed offering pointe shoe accessories that further customize the fit of a pointe shoe. Many companies, however, have resisted this trend and continue to use traditional construction techniques. Generally, pointe shoe manufacturers offer three options for dancers: introductory shoes for beginners, established production styles and sizes for intermediate and advanced students, and options in personalized alterations for advanced students and professionals.

The anatomy of today's pointe shoe includes several features. The shank—usually consisting of an inner and outer sole—lines the bottom of the foot and provides support for the arch (Figure 1). The box surrounds the toes, aiding in keeping them straight. The platform is the outer, flat tip of the box, which contacts the floor when the ballerina rises on pointe. The vamp covers the tops of the toes and foot. Often, a dancer sews on ribbons and elastics to ensure that the shoe stays on her foot. Much of this terminology evolved with pointe shoe construction and is used today to specify the style and fit of a shoe.

Coefficients of Friction of the Pointe Shoe Platform

Newtonian Mechanics and Friction

The development of pointe shoes stemmed from the desire for the ballerina to look weightless, with the ability to balance on a seemingly non-existent area. The pointe shoe's initial designs incorporated a severely tapered box and diminutive area of contact with the floor in

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[30] Judy Weiss interview by Glenna Clifton, New York City, 9 October 2009. Weiss is currently a pointe shoe fitter for Grishko, and worked with Capezio for many years.

comparison with demi-pointe (rising on the ball of the foot). The reduction of the contact area between the dancer's foot and the floor decreases the rotational traction that the dancer can utilize. With less traction, the dancer must adjust her movements to prevent slipping and avoid overpowering specific steps. Today's shoes maintain a tapered shape—though less acute—and, therefore, result in a smaller contact area when compared to "soft" ballet slippers. Almost every pointe dancer will report that, in comparison to "flat shoes," pointe shoes slip more easily and require significantly less force to turn. This qualitative observation corresponds to a quantitative alteration of the friction acting upon the dancer's shoe, measured by the coefficient of friction.

The concept of friction follows from the analysis of motion with Newtonian mechanics. Among the first, and most important, principles learned by students of physics, Newton's Three Laws of Motion define how a compact object will move when acted on by forces (in a non-accelerating frame of reference).[31] The First Law states that an object will stay at rest or in motion unless a force acts upon it. The latter may seem counterintuitive, but one must remember that all motion routinely witnessed is opposed by resistance: friction or air-resistance. On a frictionless surface or in a vacuum, an object will travel at constant velocity until a force is applied. The Second Law asserts that the net force on any body is equal to its mass multiplied by its acceleration, $F = mA$. The Third Law states that every force has an equal and opposite force, all forces are balanced. For example the force of Earth's gravity pulling on you is opposed by the force of the earth's surface pushing on you. Similarly, imagine two individuals wearing skates on ice (an approximately frictionless surface). If one skater pushes against (exerts a force on) the other, an equal and opposite force will act on the first. Both individuals will travel away from each other, but at different accelerations due to their respective masses (this follows from the

[31] Sir Issac Newton first published these laws in his Philosophiæ Naturalis Principia Mathematica in 1687.

Second Law). Newton's principles form the foundation of kinematics, the study of how objects move.

In the non-ideal world, friction influences the motion of all objects in contact with another surface and plays a crucial role in kinematics. The frictional force acts in the direction opposite of motion and exists in two forms: static and kinetic. Static friction cancels out all other forces exerted on the object, inhibiting any movement. Remember the last time you tried to push a heavy object. At first, you pushed the object without initiating any movement. This was the result of the static friction opposing your applied force. An object experiences static friction until the applied force equals the maximum static friction value, and then the object begins to move and kinetic friction takes effect. Kinetic friction works against applied forces while an object is in motion. Once an object is set into motion, one must continue to apply force to counteract the effect of the kinetic friction.

Both types of friction act in proportion to the normal force of an object—the contact force that opposes the perpendicular forces of the object acting on a surface (Figure 1). Since the normal force is usually a fraction of the gravitational force acting on its mass, the friction that acts on an object is typically proportional to the object's weight. If two people race down a snowy hill using the same sled, the heavier individual will experience a greater frictional force. The proportionality between the friction acting on an object and its normal force is referred to as the coefficient of friction and is defined by the following expressions:

$$\mathrm{F}_{Static} = \mu_s \mathrm{F}_N \qquad \text{Equation 1}$$

$$\mathrm{F}_{Kinetic} = \mu_k \mathrm{F}_N \qquad \text{Equation 2}$$

where F_{Static} is the maximum static frictional force, $F_{Kinetic}$ is the frictional force exerted on the object when in motion, μ_s and μ_k are, respectively, the coefficients of static and kinetic friction, and F_N is the normal force[32]. The magnitudes of the coefficients of friction depend on the chemical and mechanical properties of the surfaces: "both chemical adhesion between surfaces and microscopic roughness contribute to a frictional force that acts along the interface between the surfaces."[33] The expression for a coefficient of friction relates its value to physically-measurable quantities, which can be experimentally recorded and analyzed to determine the friction acting between two surfaces.

Experimental Procedure and Results

The traction of a pointe shoe platform corresponds to the friction acting between it and the flooring. To fully gauge the effect of friction, one must know the corresponding coefficient of friction. As Equations 1 and 2 demonstrate, the kinetic and static coefficients of friction can be found by experimentally determining the frictional and normal forces acting on the platform. An experiment was designed to quantify the coefficients of friction of pointe shoe platforms. In reference to Figure 1, the experiment procedure required that $F_{Applied} = 0$ and, therefore, $F_N = F_G$. The force of gravity, $F_{G,}$ was measured by holding a pointe shoe (and added weight) with a force sensor.[34] Having recorded the normal force value for the specific added weight, the force sensor was used to pull the pointe shoe at a constant velocity across the floor. From Newton's Second Law, the net force acting on an object in a given direction must equal the mass of the object multiplied by its acceleration in that given direction. Since the acceleration is equal to the change in velocity over time, pulling the pointe shoe at constant velocity makes the horizontal net force

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[32] Forces are normally measured in Newtons, N. One Newton is approximately equal to one-fourth of a pound. The coefficient of friction is unitless.

[33] Laws, Physics and the Art of Dance, 58.

[34] A PASCO PASPORT PS-2104 force sensor was used with the DataStudio computer program. The force sensor had a range of ±50 N, an accuracy of 1%, and a resolution of 0.03 N. The measurements were taken with a 10 Hz sample rate.

equal to zero, enforcing that $F_{Pull} = F_{kinetic}$ (Figure 1).[35] Therefore, the force measured by the sensor while the shoe is moving at a constant velocity represents the frictional force acting on the shoe platform. The static friction is equal to the maximum force needed to start the shoe moving (Figure 2). As demonstrated in Equations 1 and 2, these values should not rely on the platform's area and the shoe's velocity.

The procedure outlined above was applied to ten pairs of shoes. Six of the shoes were manufactured by Capezio (Aria, Contempora, and Glissé styles). The variety in the Capezio styles represented a spectrum of box sizes and platform areas. Both shoes of each pair were tested, one with a brand-new satin platform and the other with the satin scraped off of the platform (which dancers commonly do to modify their shoes). The other four pairs of pointe shoes tested were Gaynor Minden shoes of various box sizes. One shoe from each pair was tested, three with a satin platform, one with a "Suede Tip" platform. The frictional forces, kinetic and static, acting on each Capezio or Gaynor Minden shoe were recorded for five different masses added to the shoe (creating five different normal forces), three times for each mass. The experiment was performed for linoleum dance flooring and a hardwood dance floor.[36]

The data collected from the experiment was analyzed using mathematical techniques. For a given shoe, the average of the three kinetic frictional force values was plotted versus the normal force for each added mass (Figure 3). A linear regression was performed (enforcing that the y-intercept equal zero). The slope of each regression represented the kinetic coefficient of friction of the pointe shoe platform. This was repeated for each shoe style, platform type, and

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[35] The acceleration of an object is equal to the derivative of its velocity with respect to time, $\partial A / \partial t$. Therefore, when the net force of an object is equal to zero its velocity must be a constant function of time.

[36] The flooring was the Studio series manufactured by Harlequin.

floor type. The static coefficients of friction were calculated by averaging the static friction values for each mass and dividing by the normal force.

All of the kinetic and static coefficients of friction (for flooring and hardwood) were averaged, and the means and standard errors were compiled in Table 1. The average kinetic coefficients of friction on hardwood flooring were found to be 0.282 ± 0.004 for satin platforms and 0.296 ± 0.006 for canvas platforms. The average kinetic coefficients of friction on Harlequin flooring were found to be 0.428 ± 0.007 for satin platforms and 0.420 ± 0.008 for canvas platforms. For the static coefficients of friction, the hardwood values were 0.31 ± 0.01 (satin platforms) and 0.29 ± 0.02 (canvas platforms) and the flooring values were 0.45 ± 0.02 (satin platforms) and 0.42 ± 0.03 (canvas platforms). As a point of comparison, these values are in the range of the coefficient of friction between wood and leather, 0.3-0.4; the coefficient of kinetic friction between asphalt and rubber (like for the sole of an athletic shoe) is between 0.5 and 0.9.[37]

A two-way t-test[38] was used to determine if the flooring-type significantly influenced the magnitude of the kinetic or static coefficient of friction. The resulting p-values are listed in Table 1. For each pointe shoe platform type, the flooring type significantly affected the measured coefficient of friction. This proves that that flooring plays an integral role in determining a pointe dancer's traction. The frictional forces acting between pointe shoes and linoleum flooring were significantly larger than for hardwood flooring. The graphs from the experiment, however, showed no consistent trend in the effect of a satin platform versus a canvas platform or in the effect of the area of the platform. Therefore, a dancer is as likely to slip with the satin of the

[37] "Friction and Coefficients of Friction," The Engineering Toolbox, 2005, <http://www.engineeringtoolbox.com/friction-coefficients-d_778.html> (12 November 2009).

[38] The t-test is a statistical method of determining if two mean values are significantly different, not occurring due to random error. The test produces a t-value based on the means and standard deviations or each group. Using the degrees of freedom (related to the number of values averaged) of each sample group, a p-value (probability value) is determined. The p-value represents the probability that the two means are the result of random error, not significantly different. In a two-way t-test, the p-value represents the probability that one value is not significantly above or below the other.

platform ripped off or with a larger platform. Nevertheless, there was a noticeable difference in coefficients of kinetic friction between the Gaynor Minden satin platform and the Gaynor Minden "Suede Tip" platform (Figure 4).

Frictional Torque and Contact Area

The results of the experiments performed can be applied to developments in pointework across time. Pointe resulted in a significant decrease in the area of contact between a dancer's foot and the floor. On demi-pointe, the ball of the foot and the pads of the toes contact the floor. When the ballerinas of the early nineteenth century began to rise further up on their feet, they limited their contact with the floor to the tips of the toes. Lithographs from this period depict pointe shoes as severely tapered, ending almost in a point. The development of the blocked pointe shoe by Italian shoemakers in the 1860s allowed ballerinas to balance on the full platform, identifying the contact area with the sculpted size of the box. While the blocked pointe shoe may have allowed the dancer to reposition her weight, the contact area between her foot and the floor must have increased only marginally. By the early twentieth century, pointe shoe manufacturers had initiated the practice of fitting pointe shoes to the dancer's feet.[39] Consequently, boxes and platforms began to accommodate the shape of the foot. It is rumored that Anna Pavlova, the most celebrated Russian ballerina of the early twentieth century, ordered shoes with an especially broad platform and then demanded that artists retouch photos of her to show her balancing on shoes as tapered as those of the Romantic era.[40] The customization of pointe shoes by manufacturers produced a greater contact area between the dancer's foot and the floor. The box and platform of the shoes continued to widen gradually throughout the twentieth century,

[39] Attfield, 25.
[40] Eliza Minden interview by Glenna Clifton, New York City, 9 October 2009.

increasing considerably during the 1970s.[41] Pointe shoes today come in a variety of box shapes and sizes.

Ideally, a change in contact area should not influence the coefficient of friction. Nevertheless, since the coefficient of friction is a measurement of the resistance produced from the contact between two uneven surfaces, a decrease in contact area will most probably result in a smaller coefficient of friction. While further experimentation is needed to confirm this hypothesis, the relation between contact area and friction may have affected the frictional forces acting on the pointe shoe.

Frictional Torque and Pirouettes on Pointe

The main influence of contact area involves the frictional torque acting on a pointe shoe. Torque arises when a force is exerted on an object away from its center of mass. For example, the force of a child sitting on the end of a seesaw induces rotation of the bench around its fulcrum (pivot point). In non-calculus based physics, torque is defined as the exerted perpendicular force multiplied by the length of the lever arm (the distance between the force exertion and the point at which the torque is being measured):

$$\tau = r\mathrm{F}_{\perp} \qquad \text{Equation 3}$$

where τ is the torque (in Newton-meters), r is the lever arm length, and $F_{\perp}$ is the force perpendicular to the lever arm (Figure 6).[42]

As described in the seesaw example, an unopposed torque produces rotation, also known as angular momentum, L. This value is easier to understand by relating it to linear momentum. If two cars crash, the more massive automobile will most likely incur less damage. This is a result

[41] Weiss interview.

[42] The torque of an object is technically defined as the curl product of the applied force and the lever arm: $\vec{\tau} = \vec{r} \times \vec{F}$.

of the difference in linear momentum of the two cars. Linear momentum, *p,* is defined as the mass of an object multiplied by its velocity ($p = m*v$); it represents the effort needed to stop an object from moving. A larger car will be harder to stop than a smaller car traveling at the same speed since the larger car has a greater linear momentum. A car traveling slowly will be easier to stop than a faster car of the same size since the slower car has less linear momentum.

In a similar manner, angular momentum can be thought of as describing the effort needed to stop an object from rotating. Angular momentum, L, is defined as the product of an object's angular mass (its moment of inertia, I) with its angular velocity, ω:

$$L = I\omega \qquad \text{Equation 4}$$

The moment of inertia represents the bulk of the object: the distribution of its mass in relation to the axis of rotation. A book has a greater moment of inertia than a pipe since its mass is distributed farther from any possible axis of rotation. The angular velocity measures how many degrees of rotation the object travels through per unit of time, in other words, how fast the object is spinning. Since the angular momentum is equal to the product of the moment of inertia with the angular velocity, the angular momentum of the pipe could equal or exceed that of the book if the difference in angular velocity compensates for the disparity in the moments of inertia.

As previously stated, angular momentum is produced from unopposed torque acting on an object:

$$\tau = \frac{\Delta L}{\Delta t} = \frac{dL}{dt} \qquad \text{Equation 5}$$

where τ is the torque, and dL/dt is the change in angular momentum over a period of time. Any change in angular momentum stems from a torque acting on the object. When a dancer prepares for a turn, she produces a torque on her body by using the separation of her feet to exert forces at distances away from the axis of rotation. The angular momentum of her turn depends on the magnitude of the torque and the length of time that it is exerted.[43] While the angular momentum of a spinning object remains constant, the rate of the turn can vary depending on the position of the body.[44] This is commonly observed in ice skating: a turn will speed up when a skater pulls the arms in.

Unlike ice skaters, however, ballet dancers do not turn on an approximately frictionless surface. A dancer experiences a decelerating torque produced by friction between the foot and the floor. This causes a pirouette (a turn) to slow to a stop after a finite number of revolutions. The frictional force can be thought of as acting equally on each point of the contact area. Therefore, a larger contact area increases the torque (the force multiplied by the distance from the axis of rotation). Using the coefficients of friction of today's pointe shoe platforms, one can calculate the torque acting on the dancer over a single rotation. From this, one can determine the number of rotations that a dancer, with a given initial angular momentum, will complete before slowing to a halt. This computation will illustrate the impact of the pointe shoe platform shape and size.

To calculate the torque, the pointe shoe platform dimensions must first be determined. Using the donated shoes from Capezio and Gaynor Minden, the size of each platform was measured.[45] To demonstrate the range in torque affecting a dancer, the dimensions of the smallest and largest platform were used. The smallest platform was approximately an ellipse with horizontal length, 2a, equal to 3.2 cm and vertical length, 2b, equal to 2.1 cm. The largest platform had parameters 2a = 5.0 cm and 2b = 3.0 cm. Since all of the shoes produced similar

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[43] L. Lei, K. Laws, and T. Zhen, "Angular Momentum in Dance Turns: The Zi Tui Fan Shen," Kinesiology and Medicine for Dance , 14(2) (1992): 58.
[44] Kenneth Laws, "An Analysis of Turns in Dance," Dance Research Journal 11(1/2) (1978-1979): 14.
[45] The platform of one shoe from each pair was covered in paint and then pressed onto paper with a full-body force.

coefficients of friction, the average was used. The rotational motion of the shoe requires the use of the kinetic coefficient of friction (0.294 for hardwood and 0.424 for flooring).

Using the expressions for kinetic friction and torque (Equations 2 and 3), the torque over one revolution was calculated for the smallest and largest platforms on hardwood and flooring (Figure 7). For the smaller platform the torque was found to be 1.46 N*m on hardwood and 2.11 N*m on linoleum. For the larger platform the torque was found to be 2.34 N*m on hardwood and 3.38 N*m on linoleum. Even from these intermediary calculations, one can see that the larger platform induces a greater resistive torque. Since the torque acting on a pointe shoe platform causes a decrease in angular momentum, a larger torque will more efficiently oppose the rotation of a dancer and slow her pirouette:

$$\tau = \frac{dL}{dt} = \frac{d}{dt}(I\omega) \underset{I = \text{const}}{=} I\frac{d\omega}{dt} = I\alpha \qquad \text{Equation 6}$$

where L is the angular momentum, I is the moment of inertia, ω is the angular velocity, and α is the angular acceleration. If the moment of the inertia remains approximately constant (the distribution of the dancer's mass does not change), the torque will equal the moment of inertia multiplied by the angular acceleration.

In order to solve for the number of rotations that a dancer can complete on pointe before slowing to a stop, we must know the values of a typical dancer's mass, angular velocity and moment of inertia. According to tests and analysis performed by Kenneth Laws, the average dancer has a mass of 50 kg, a spinning rate of 2 full revolutions per second (12.6 radians/sec) in en dehors pirouettes, and a moment of inertia of 0.5 kg*m^2 when in the pirouette position.[46] For these values, the dancer has an angular momentum of 6 kg*m^2/s^2. Thus, the deceleration of the dancer on hardwood flooring is calculated as -2.92 rad/s^2 for the small platform and -4.68 rad/s^2

[46] Laws, "An Analysis of Turns in Dance," 16.

for the large platform (Figure 8). On linoleum flooring, the deceleration is -4.21 rad/s^2 for the small platform and -6.75 rad/s^2 for the large platform. The deceleration corresponds to the decrease in the dancer's rotational speed per second. This means that a dancer on hardwood flooring will be able to complete a maximum of 4.14 turns in 5.32 seconds with the smaller platform and a maximum of 1.94 turns in 4.12 seconds with the larger platform. On linoleum dance flooring, a dancer should be able to complete a maximum of 2.39 turns in 4.34 seconds with the smaller platform and a maximum of 0.56 turns in 3.43 seconds with the larger platform.

In his experiments to determine the angular momentum of the average dancer, Laws studied a professional dancer performing pirouettes on a wooden platform. Assuming that her pointe shoes incorporated a medium-sized platform, this paper's results suggest that she should have been providing enough force to slow after 3 turns (assuming she maintained her balance). Since it is generally preferred that pirouettes slow almost to a stop before the dancer puts down her working leg, one can conclude that the studied dancer would have provided enough force to complete two or three revolutions before stopping. Therefore, the results of this study complement Laws' findings.

Application of Frictional Toque to Pointework throughout Time

As the previous results reveal, the size of the platform plays an important role in limiting the torque that acts on a pointe shoe. If we assume that the pointe shoe satin used during the eighteenth and nineteenth centuries had similar coefficients of friction as the satin used today, we can apply these results to the development of pointe technique. The desire for tapered pointe shoes during the Romantic era restricted the torque that ballerinas could utilize on pointe, requiring technical adjustments to prevent overpowering steps (like turns). The decrease in rotational traction between the dancer's pointe shoe and the floor also required her to turnout with little aid from the floor. The reduced frictional resistance against the floor resulted in the need for the dancer to augment her strength and control.

With the gradual increase in box width and platform area during the twentieth century, dancers were able to utilize the greater oppositional frictional torque to increase turnout during turns and other steps on pointe. Starting in the late 1950s, George Balanchine required that all of his female dancers wear pointe shoes in every technique class.[47] He also wanted a more consistent use of 180 degree turnout in a growing number of positions. His choreography emphasized strength, articulation, and speed. The wider platforms of the time would have provided Balanchine's dancers with greater traction with the floor, allowing them to utilize this resistance to increase their turnout. Additionally, in order to complete the same number of turns, a dancer with a large pointe shoe platform must provide a greater initial force than a dancer with a small platform (Figure 8). This corresponds to a greater initial angular velocity, which results in the appearance of faster pirouettes. Furthermore, a dancer with a large platform must balance for a longer amount of time to perform the same number of turns, requiring greater strength. Therefore, the increase in pointe shoe platform size could have contributed to Balanchine's choreographic advances (turnout, speed, and strength) by allowing his dancers to exploit the frictional torque acting on their shoes.

In 1964, the New York State Theater opened with a stage designed to Balanchine's specifications.[48] Among these was the use of a linoleum sprung floor. Before this time, dancers performed on hardwood or less desirable flooring (including concrete or gymnasium flooring). Many ballet companies faced the challenge of dancing on hard, uneven, and slippery floors while

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[47] Schorer, 6.
[48] Schorer, 7.

on tour.[49] As shown through the experiments presented in this study, dance flooring significantly increases the friction that acts upon pointe shoes. By implementing the use of linoleum flooring, Balanchine assured that his dancers received enough traction to fully turn out their legs. After the theater opened, dance flooring companies began to offer flooring options for studios and performing venues.[50] The proliferation of flooring allowed schools and companies to provide level, non-slippery dance floors for their dancers, regardless of the venue.

Today's pointe shoes include significantly larger boxes than those from even forty years ago. Despite this increase, pointe shoes still result in a smaller contact area than for demi-pointe. The increased traction that the larger platform, along with the linoleum dance flooring, creates may well contribute to the extreme turnout achieved by dancers today. We also see an increase in the number of turns that dancers are able to complete. This technical development is not substantiated by the findings of this study, however, and probably relates to how wider platforms facilitate balancing, allowing for more revolutions.

Other Factors Affecting Frictional Torque

Several other factors could have contributed to the frictional forces acting on pointe shoes throughout their development. As previously mentioned, changes in the construction materials of pointe shoes could have altered the torque exerted on the platform. The custom of covering the outside of the pointe shoe with satin has persisted over time, ever since its known use in the early nineteenth century. The quality of fabric available, however, has probably improved in durability. According to Dan Terlizzi, Company Director of Capezio, the satin they use today is much improved since the late nineteenth century when the company was founded.[51] The older satin may have been more or less slippery, but it could also have frayed more easily, producing a

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[49] Rubinson interview.
[50] Rubinson interview.
[51] Terlizzi interview.

rougher surface (and more friction). No tests have been conducted comparing specific satin types from today and the past.

Flooring also plays a significant role in increasing the coefficient of static and kinetic friction. As was previously discussed, today's dance floorings considerably increase the frictional forces acting on the pointe shoe. In the past, though, dancers would adapt their pointe shoes in order to maximize their traction on slippery wood floors. Darning was commonly used for supportive and protective purposes during the nineteenth century. The roughness of the treated platform also probably augmented the friction acting on the pointe shoe. The use of darning continues today, although its popularity has considerably diminished. As in the past, many dancers today scrape the satin off of the platforms of their shoes to reveal the canvas lining. As this study's results show, this tactic does not significantly affect the coefficient of friction. It may be the case that the canvas lining picks up dirt or rosin easier and, therefore, enhances its traction over time. The shoes used in this experiment were brand new, making it impossible to test this hypothesis.

Rosin is another method that dancers use to increase friction. Additionally used by string-instrument players, rosin has the unique characteristic of a high static coefficient of friction and a low kinetic coefficient of friction.[52] Therefore rosin prevents slipping while allowing kinetic motion. Dancers may also wet their shoes to increase the coefficient of friction. Water, in small quantities, increases the friction between two surfaces, but in large quantities produces a slippery layer of liquid.[53] Dancers use all of these adaptations to increase traction with the floor. How dancers altered their pointe shoes in the past may well have influenced the frictional torque

[52] Laws, Physics and the Art of Dance, 58.
[53] Laws, Physic and the Art of Dance, 59.

affecting pointework; however, the variations are so great than any definite conclusion would lack consistency.

Future Implication of the Physical Analysis of Pointe Shoes

Relative to the changes in pointe technique that have taken place over the last 100 years, pointe shoe manufacturing methods have remained relatively static. Most companies continue to use lasts to create “turned” pointe shoes. Capezio, Grishko, and Freed, three of the largest manufacturers of pointe shoes, use this method. Each of these companies started as a group of “makers”—individuals who constructed a shoe in their specific style. In the 1980s, Capezio switched to an assembly line process.[54] They continue to make individualized shoes, called Special Make-Ups (SMUs), by hand. Freed has retained the practice of individual makers constructing full pairs of pointe shoes.[55] Each Freed maker constructs the pointe shoe box in a specific shape. When a maker retires, so does the box shape. While each shoe is not necessarily made for a single wearer’s foot, each pair of shoes is constructed by a single person. These companies take pride in the tradition and character of their manufacturing techniques and aim to preserve the authenticity of construction methods.

Some companies have incorporated new materials into their shoes while others assert that their materials define their shoes. Capezio has utilized advances in satins, threads, and glues. Originally, each Capezio maker constructed the box by layering burlap, newspaper, and fish paste (a mixture of ground-up fish scales with flour and water).[56] This paste would release a horrible odor when exposed to moisture. Changes in the recipe for Capezio glue resulted in a less

[54] Terlizzi interview.
[55] Brenda Neville telephone interview by Glenna Clifton, 6 October 2009. Neville is a head pointe shoe fitter for Freed of London and previously worked with Gaynor Minden.
[56] Terlizzi interview.

pungent construction material. Capezio has also experimented with different platform coverings. The Dura Toe replaces the satin platform and pleats with suede. Freed, on the other hand, has maintained the same materials from when the company was founded. According to Freed, the materials allow the shoe to mold to the foot and break in properly.[57]

While dedicated to preserving tradition, all pointe shoe companies continue to develop new products. The process of testing current products, designing new shoes, and testing new shoes varies by company. Product development in most companies focuses on the shape of the pointe shoe. Capezio uses the preferences of professionals to create new styles.[58] When designing a new style, the company uses professional dancers to test the shoes and give feedback about the changes. Freed also uses this method, but supplements it with some scientific testing of current products. This testing takes place in London, where the company is centered, and involves testing the endurance of the shoe.[59] Like Capezio, Freed conducts qualitative trials of new pointe shoes.

One company, however, vastly differs from Capezio, Freed, and Grishko in construction method, materials used, and new product development. Gaynor Minden was founded by Eliza Minden in 1993 in the hope of applying some of the revolutionary advances in material science to pointe shoe construction. Her patented design includes a fused box-shank made out of an elastomeric (a material similar to that used in scuba fins).[60] She has incorporated shock absorbing cushioning (Poron cellular eurothane foam) in her shoes to aid the dancer and reduce the noise of the shoe. In designing her shoe, Minden consulted with several dance-related doctors

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[57] Neville interview.
[58] Terlizzi interview.
[59] Neville interview.
[60] Minden interview.

and scientists. When investigating a material, she would make a prototype and test it personally and then in larger numbers.[61]

Some dancers and dance teachers dislike Gaynor Minden shoes on the grounds that pointe shoes should not help the dancer complete steps, like the relevé. This viewpoint explains why the utilization of science in shoe construction was so much slower than in other athletic arenas. Moreover, Eliza Minden explains:

> Dancers are so disciplined; they are trained not only not to complain, but not to speak. So the last thing that they're going to do is to complain about their toes. Also, the pointe shoe has become almost mythologized into a symbol and this rite of passage: yes it's going to hurt; but you're an artist and you have to suffer.[62]

The painful blisters and bunions that result from pointework are just another part of the job for dancers. The difficulty of the art form intensifies the satisfaction in perfecting it. As athletes, however, dancers routinely require treatment for injuries. Perhaps the integration of scientific research could produce safer shoes while meeting the artistic demands of the dance world.

Although Eliza Minden consulted many scientists when designing her shoe, only a few studies have been conducted on the physical implications of its use. In the research conducted by Bryan Cunningham and others, various brands of pointe shoes were tested for stiffness and strength in the box and shank. Their results showed that Gaynor Minden pointe shoes only fatigue after ten times more cycles (in their study, simulated relevés) than the other brands.[63] Another study revealed that Gaynor Minden shoes produce better ankle alignment, even for highly-trained dancers.[64] The results from the experiments presented in this paper show that the shape and material of the pointe shoe platform significantly affects the frictional rotation that a

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[61] Minden interview.
[62] Minden interview.
[63] Cunningham et al., 558.
[64] Lise Worthen ,"A Kinematic Analysis of Internal/External Rotation at the Knee Joint during Two Ballet Movements," Journal of Dance Medicine Science, 2(4) (1998): 153.

dancer can utilize. By performing further prognostic tests, pointe shoe manufacturers could develop a platform that aids the dancer's turnout and turning. Any number of other scientific studies could determine the most successful designs of pointe shoes. Combined with an adherence to the aesthetic desires of the dance community, pointe shoes manufacturers could continue to develop new pointe shoes (in materials and design) that support changes in pointework technique while also reducing injuries.

I would like to extend a special thank you to Lynn Garafola and Timothy Halpin-Healy for all of their invaluable advice and support. Additionally, I'd like to thank Dan Terlizzi, Dawn Terlizzi, Eliza Minden, Judy Weiss, Rhonda Rubinson, and Kathryn Sullivan for their contributions to my project. Lastly, I'd like to thank Kenneth Laws for his inspiration.

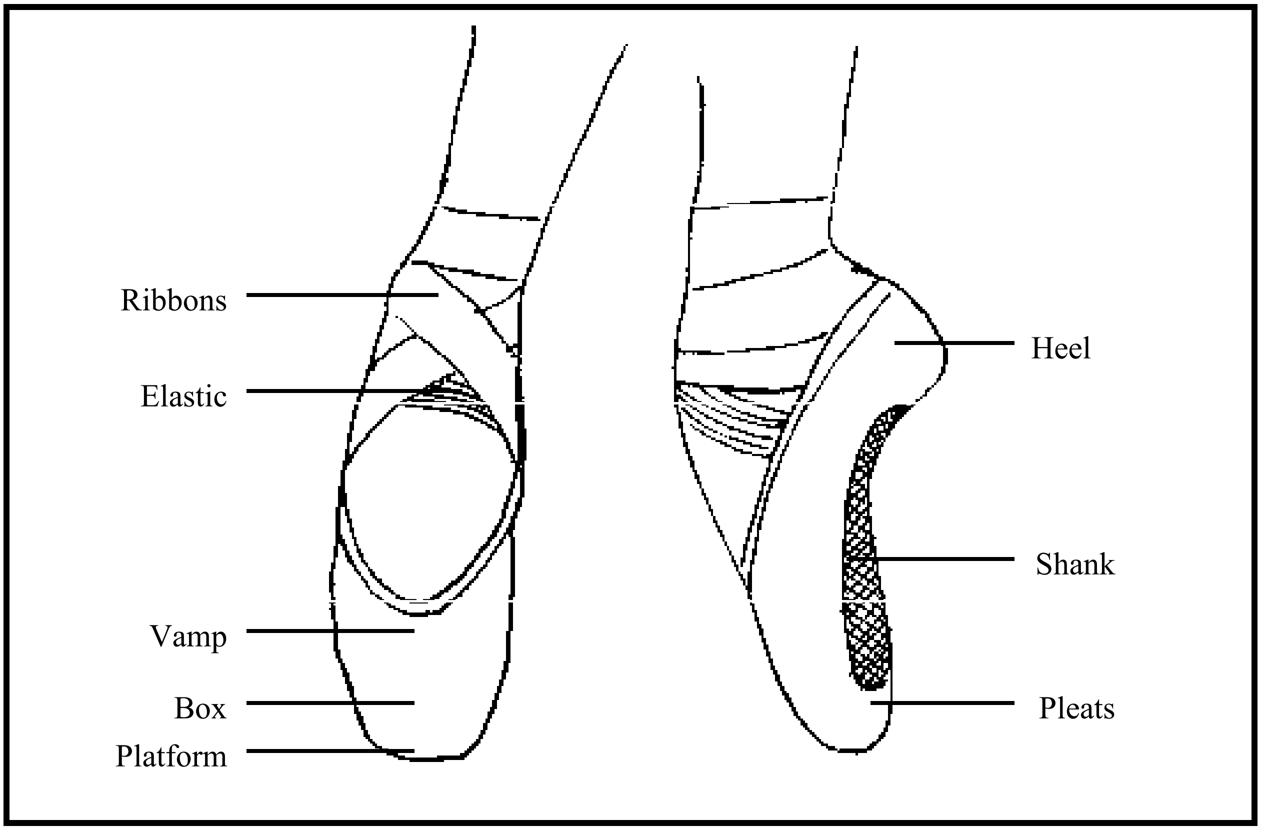


Figure 1 – This drawing shows the common names for parts of a pointe shoe. These traits acquired their form and names over time, but most date back to the beginning of the twentieth century.

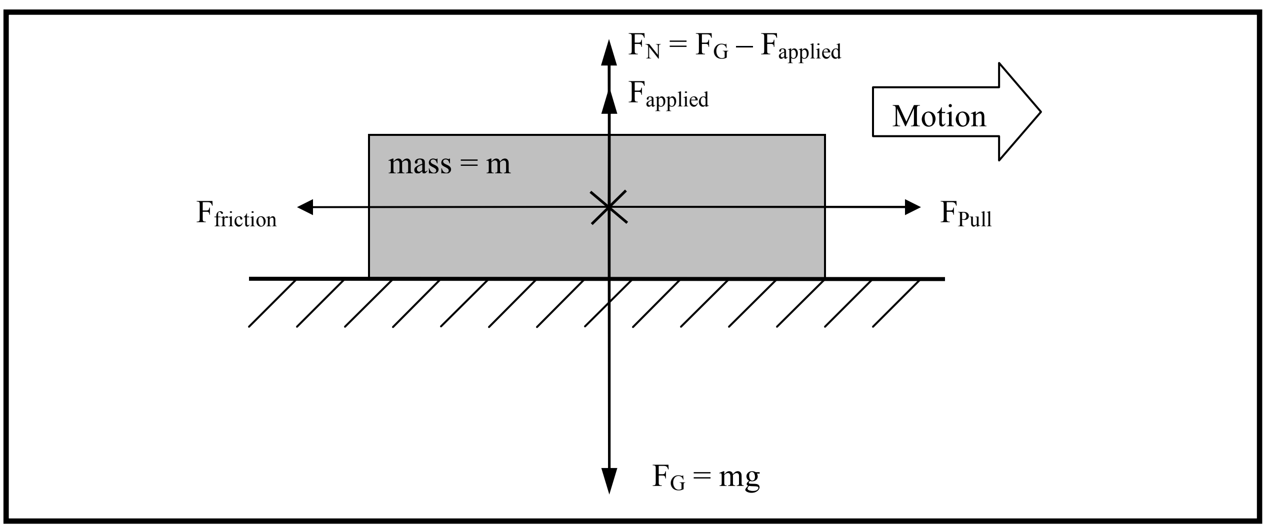


Figure 2 – Free body diagram of the forces acting on an object in contact with the floor. All forces act upon the object at its center of mass, the X. The force of gravity, F_G, is equal to the object's mass multiplied by the gravitational constant. From Newton's Third Law (in the vertical direction) the normal force, F_N, is equal to any applied force subtracted from the gravitation force, F_G-$F_{applied}$. For pulling the object at constant velocity in the horizontal direction, Newton's Second Law reveals that the force used to pull the object must be equal to the kinetic frictional force acting on the object, F_{Pull}=$F_{friction}$.

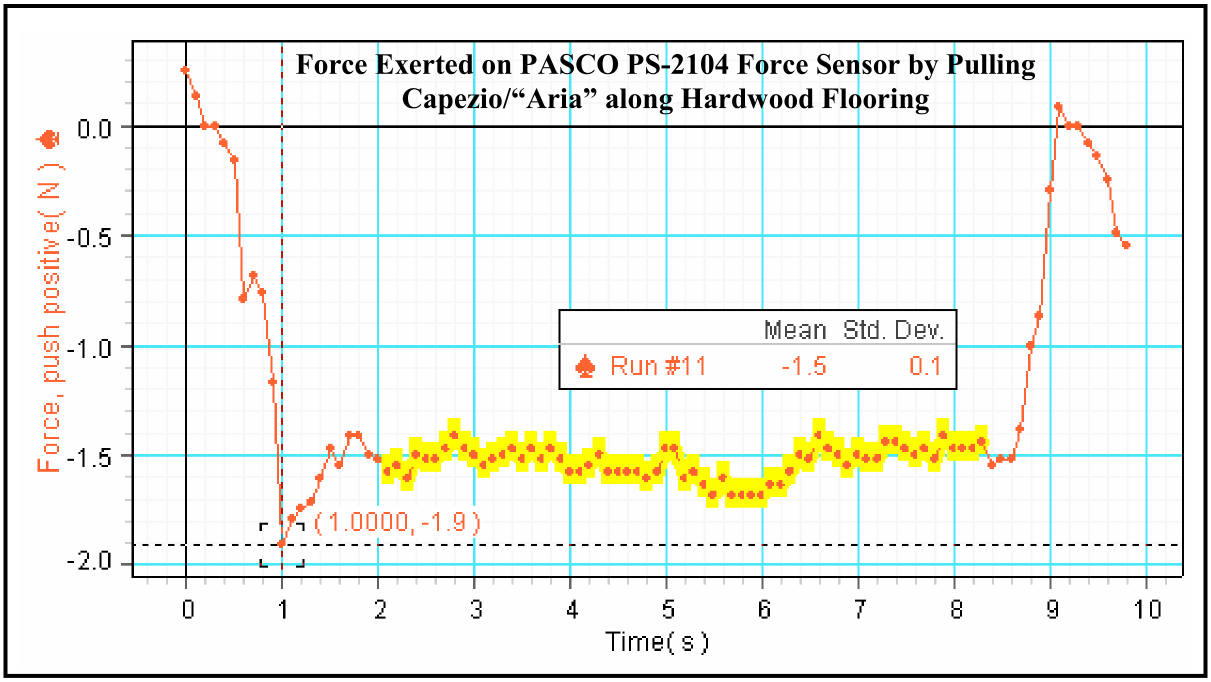


Figure 3 – Measurement of the frictional force (N) acting on a Capezio/ "Aria" pointe shoe pulled at constant velocity on hardwood flooring. The recorded values are negative since the software measured pulling forces as negative. The spike at 1 second shows the maximum force needed to overcome static friction. The coefficient of friction is calculated from the maximum frictional force, that at t =1s. The highlighted area shows the data recorded with the shoe pulled at approximately constant velocity. The mean and standard deviation of the highlighted data is showed on the graph.

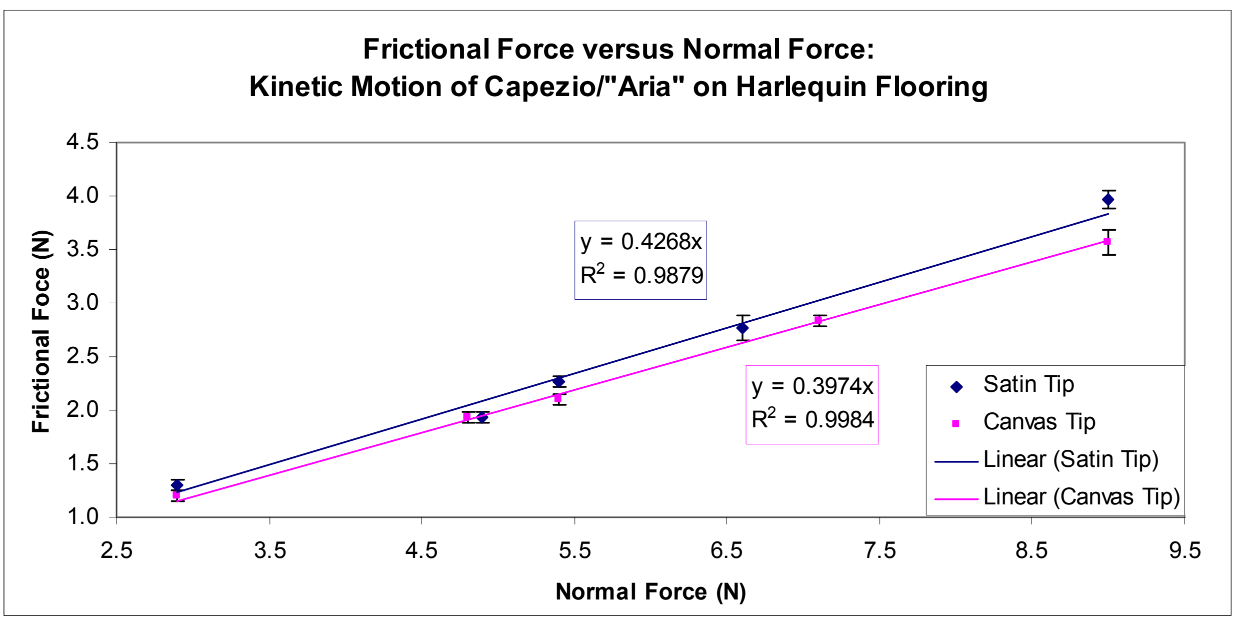


Figure 4 - Frictional force versus normal force graph for the platform of a Capezio "Aria" pointe shoe pulled at constant velocity across Harlequin Studio Flooring. The "Satin Tip" shoe (blue) had a platform covered in satin. The "Canvas Tip" shoe (pink) had a platform whose satin had been removed, leaving the canvas lining. The error bars represent the calculated error for measuring and averaging each frictional force value. Linear regressions were fit to each data set, enforcing that the y-intercept equal 0. The slope values, equivalent to the coefficients of kinetic friction, and goodness-of-fit values (R^2) of each regression are displayed on the graph.

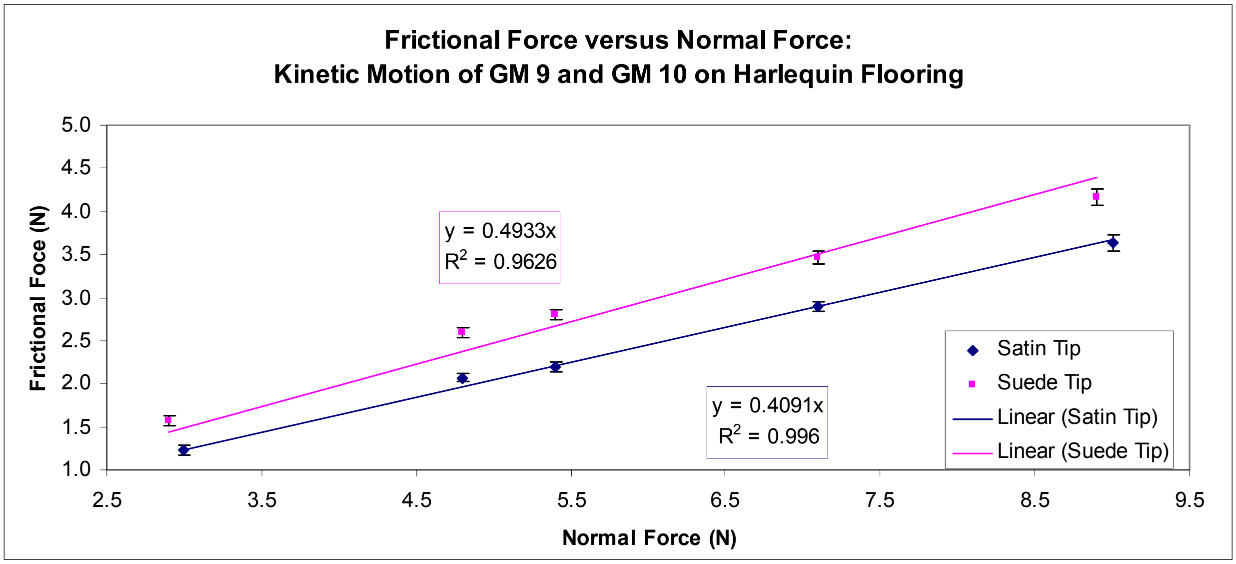


Figure 5 – Frictional force versus normal force graph for the platform of a pointe shoe pulled at constant velocity across Harlequin Studio Flooring. The "Satin Tip" shoe (blue) and the "Suede Tip" shoe (pink) were the same style and size of shoe made by Gaynor Minden. The error bars represent the calculated error for measuring and averaging each frictional force value. Linear regressions were fit to each data set, enforcing that the y-intercept equal 0. The slope values, equivalent to the coefficients of kinetic friction, and goodness-of-fit values (R^2) of each regression are displayed on the graph.

KINETIC								
PLATFORM	HARDWOOD		LINOLEUM		BOTH		T-VALUE	P-VALUE
	AVE	SE	AVE	SE	N	DF		
Satin	0.292	0.004	0.428	0.007	6	8	16.186	0.000000
Canvas	0.296	0.006	0.420	0.008	6	8	12.626	0.000001
STATIC								
PLATFORM	HARDWOOD		LINOLEUM		BOTH		T-VALUE	P-VALUE
	AVE	SE	AVE	SE	N	DF		
Satin	0.31	0.01	0.45	0.02	6	8	5.148	0.000877
Canvas	0.30	0.02	0.42	0.03	6	8	4.047	0.001860

Table 1 – Kinetic and Static Coefficients of Friction between Hardwood or Dance Flooring and Pointe Shoes with Satin or Canvas Platforms. The above table shows the calculated average values (AVE) and standard errors (SE) of the kinetic and static coefficients of friction on hardwood and Harlequin dance flooring. The values for shoes with a satin platform and for those with a canvas platform are noted. The number of members (N) of each sampling group and the corresponding degrees of freedom (dF) are listed. Using these values, a two-way t-test was performed resulting in the listed t-values. The corresponding p-values determine if the two averages are significantly different (p-value < 0.05). Based off of the resulting p-values, the coefficients of friction are significantly affected flooring-type.

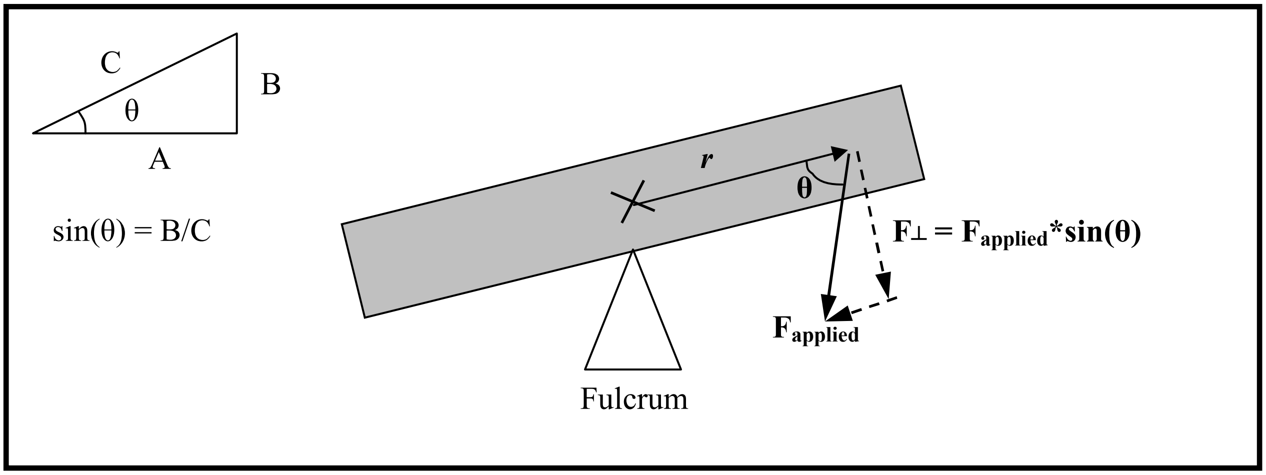
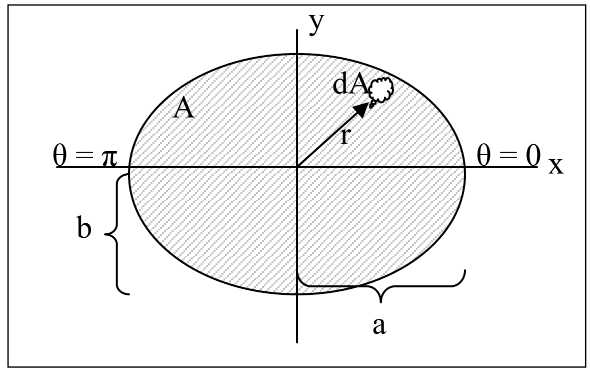


Figure 6 – Torque acting around a fulcrum. The torque of the rectangle around the fulcrum (the white triangle) is equal to the lever arm, r, multiplied by the perpendicular applied force (from the components of the vector of the applied force). From sinusoidal properties, the perpendicular force is equal to the applied force multiplied by sine of the theta.

$$\tau = rF$$

$$\tau = \int d\tau = \int r(dF)$$

$$F = \mu_k N$$



For the Normal force acting on an infinitesimal section of the platform, dA:

$$N = \frac{Mg}{A} dA$$

$$\int \tau = \frac{\mu_k Mg}{A} \iint_{\text{Area of Platform}} r\,dA$$

Equation of an ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$y = \pm b\sqrt{1 - x^2/a^2}$$

$$x = a\cos(\theta)$$

$$y = b\sin(\theta)$$

$$x^2 + y^2 = R^2$$

$$R^2 = a^2\cos^2(\theta) + b^2\sin^2(\theta)$$

$$\int d\tau = \frac{4\mu_k Mg}{A}\int_0^{\pi/2}\int_0^R r^2 dr d\theta = \frac{4\mu_k Mg}{A}\int_0^{\pi/2}\left[\frac{1}{3}R^3\right]d\theta$$

$$\int d\tau = \frac{4\mu_k Mg}{3A}\int_0^{\pi/2}\left[a^2\cos^2(\theta) + b^2\sin^2(\theta)\right]^{3/2} d\theta\tau$$

$$M = 50\,kg$$

$$g = 9.81\,m/s^2$$

$$\pi \approx 3.14$$

$$A = \pi ab$$

	HARDWOOD		**FLOORING**	
μ_k	0.294		0.424	
PLATFORM	**SMALL**	**LARGE**	**SMALL**	**LARGE**
a	.016 m	.025 m	.016 m	.025 m
b	.011 m	.015 m	.011 m	.015 m
A	0.00053 m^2	0.00118 m^2	0.00053 m^2	0.00118 m^2
τ	1.4609 N*m	2.3404 N*m	2.1068 N*m	3.3753 N*m

Torque acting on the shoe at any point in time, opposite to the direction of rotation.

Figure 7 – The above derivation shows the use of a double integral to determine the torque acting on a pointe shoe platform with given normal force (N), coefficient of friction (μ_k), and dimensions (a and b). The normal force can be rewritten as dN, the normal force acting on a infinitesimally small section of the platform. Integrating over the ellipse for dN multiplied by r results in the expression for determining the torque of the pointe shoe platform.

Calculation of number of turns possible with different shoes, given average angular momentum.

Given :

$M = 50 \text{ kg}$

$I = 0.5 \text{ kg} * \text{m}^2$

$\omega_0 = 12.6 \text{ rad/s}$

$$\tau = I\alpha$$

$$\alpha = \frac{\tau}{I}$$

$$\omega_f^2 = \omega_0^2 + 2\alpha t^2$$

$$t = \sqrt{\frac{\omega_f^2 - \omega_0^2}{2\alpha}}$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\text{rotations} = \frac{\theta}{2\pi}$$

	HARDWOOD		**FLOORING**	
μ_k	0.294		0.424	
PLATFORM	**SMALL**	**LARGE**	**SMALL**	**LARGE**
τ	1.4609	2.3404	2.1068	3.3753
deceleration, α	-2.9217 rad/s^2	-4.6808 rad/s^2	-4.2137 rad/s^2	-6.7506 rad/s^2
time until stop, t	5.212 s	4.118 s	4.340 s	3.429 s
angle until stop, θ	25.986 rad	12.198 rad	15.000 rad	3.517 rad
rotations until stop	**4.135**	**1.941**	**2.387**	**0.560**

Figure 8 –Using the torques calculated in the previous figure, this figure shows the calculations of the number or rotations until the torque of the pointe shoe platform forces the dancer to stop turning. The given values of the dancer's mass, moment of inertia, and angular velocity were determined by Kenneth Laws in his article, "An Analysis of Turns in Dance." The moment of inertia was approximated for a dancer in the pirouette position, with one leg bent, the foot resting the knee of the standing leg. The angular velocity was calculated from measuring the rotational speed of a professional dancer's *en dehors* pirouettes on a wooden platform.

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